SPECTRAL DIRECTIONAL AND HEMISPHERICAL EMISSIVITIES OF A PLANE ANISOTROPICALLY SCATTERING LAYER

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The influence of anisotropy of volume scattering on the emissivity of a semiopaque plane layer is analyzed.

The majority of heat-shielding materials utilized in modern engineering are semiopaque to thermal radiation. The experimental determination of the emissivities of such materials is made complicated by the need to take account of the bulk nature of the emission and radiation of surrounding bodies. In this connection, theoretical computations that are most often based on the solution of the transport equations in substantially simplified formulations relative to the geometry [1, 2] or properties of the semiopaque medium [2-7] acquire great value. Taking account of volume scattering results in considerable difficulties in the solution of the transport equations. Such an analysis is carried out in [8, 9], say for a semiopaque isotropically scattering plate.

The purpose of this paper is to investigate the influence of the volume scattering index, multiple reflection, and optical thickness of a layer on the nature of the change in and the magnitude of the spectral directional $\varepsilon_\nu(\Theta)$ and spectral hemispherical $\tilde{\varepsilon}_\nu$ emissivities of a plane semiopaque layer to thermal radiation.

To determine the emissivities of a semiopaque layer it is necessary to know the thermal radiation intensity distribution therein. The radiation intensity was analyzed under the following assumptions. It was considered that interference and radiation polarization effects are negligibly small; the characteristic time scales are significantly greater than d/c; a local thermodynamic equilibrium is set in the layer; the layer surfaces are optically smooth and partially penetrable for thermal rays; the scattering particles of the medium are homogeneous, isotropic, and possess spherical symmetry; the radiation intensity is independent of the azimuthal angle.

The radiation transport boundary-value problem was written as follows [10]:

$$\pm\cos\Theta\frac{\partial\Phi_\nu^\pm}{\partial x}=-\alpha_\nu\Phi_\nu^\pm+\frac{\beta_\nu}{2}\int_0^{\pi/2}J(\Theta',\ \Theta)\,\Phi_\nu^\pm\sin\Theta' d\Theta'+B_\nu(T)\,\varkappa_\nu n_\nu^2/\pi;$$

$$\Phi_\nu^+(0,\ \Theta)=R_2\Phi_\nu^-(0,\ \Theta)+\Phi_{\nu,e}^+(\Theta)(1-R_1)n_\nu^2; \qquad (1)$$

$$\Phi_\nu^-(d,\ \Theta)=R_2\Phi_\nu^+(d,\ \Theta)+\Phi_{\nu,e}^-(\Theta)(1-R_1)n_\nu^2.$$

According to the assumptions made in formulating the problem, the scattering index depends only on the angle Θ_0 between the directions of the incident Θ' and scattered Θ rays and is a surface of rotation with axis Θ'. The layer emissivities were determined by means of the formula

$$\varepsilon_\nu(\Theta)=\Phi_\nu(0,\ \Theta)/B_\nu(T);\quad \tilde{\varepsilon}_\nu=2\int_0^{\pi/2}\varepsilon_\nu(\Theta)\sin\Theta\cos\Theta d\Theta.$$

The system (1) was solved numerically. A detailed description of the algorithm and examples of the computations are presented in [11, 12].

The computation of $\varepsilon_\nu(\Theta)$ and $\tilde{\varepsilon}_\nu$ was performed for an isothermal layer with temperature T = 1473°K and $\varkappa_\nu$ = 100 m^{-1}. The frequency $\nu = 1.5251\cdot10^{14}$ sec^{-1} corresponding to the Planck distribution maximum for T = 1473°K was selected. The layer optical thickness τ was varied within the limits $0.5 \leqq \tau \leqq 5$, and the albedo within $0.25 \leqq \omega \leqq 0.989$. The influence of

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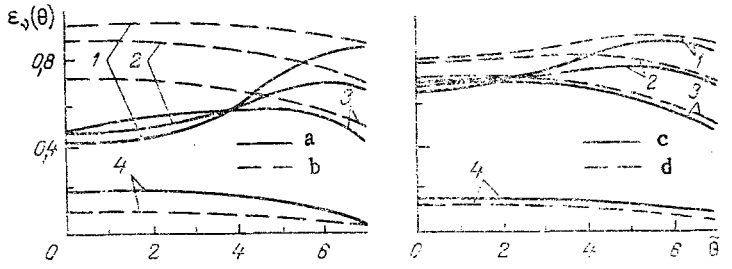

Fig. 1. Influence of the albedo (ω) and optical thickness of the layer (τ) on the nature of the dependence of the magnitude of the spectral directional emissivity of a nonrefracting isotropically scattering layer on the angle $\tilde{\theta}$ [1) ω = 0.25; 2) 0.5; 3) 0.75; 4) 0.989]: a) τ = 0.5; b) 5; c) 1; d) 1.5.

anisotropy of the bulk scattering on the layer emissivities was studied for three kinds of index

$$J(\Theta_0) = 1; \tag{2}$$

$$J(\Theta_0) = 1 + p\cos\Theta_0; \tag{3}$$

$$J(\Theta_0) = 1 + p(\cos\Theta_0)^3. \tag{4}$$

The results presented in this paper are obtained for p = 1.

It is convenient to represent the radiative characteristics of an anisotropically scattering layer in comparison with the analogous quantities of an isotropic layer. Consequently, in place of $\varepsilon_\nu(\Theta)$ and $\tilde{\varepsilon}_\nu$ the relative quantities

$$\eta(\Theta) \equiv \frac{\varepsilon_\nu^1(\Theta) - \varepsilon_\nu^0(\Theta)}{\varepsilon_\nu^0(\Theta)} \cdot 100\%; \qquad k \equiv \frac{\tilde{\varepsilon}_\nu^1 - \tilde{\varepsilon}_\nu^0}{\tilde{\varepsilon}_\nu^0} \cdot 100\%$$

were studied in cases of anisotropic scattering. Specific values of $\varepsilon_\nu{}^1(\Theta)$ and $\tilde{\varepsilon}_\nu{}^1$ can be obtained by using Figs. 1-3, where results are presented of a computation of the radiative characteristics of an isotropic layer, Table 1, in which the quantities $\eta(\Theta)$ are represented, and Fig. 4 where the dependences $k(\tau, \omega, n)$ are displayed graphically.

Analysis of the results obtained for an isotropic layer resulted in the following deductions. Scattering exerts influence on the energy distribution in the direction Θ in two ways (Fig. 1). On the one hand, because of the contribution of other directions, its increase occurs, and on the other hand it diminishes because of scattering in other directions. These processes result in the fact that a sufficiently thin layer in the direction of small Θ has a low emissivity due to the small effective radiation thickness. In this case the energy scattering in the other directions will be significantly less than the energetic contribution of the directions having a high effective radiating length. Consequently, the quantity $\varepsilon_\nu(\Theta)$ grows for small Θ of the scattering layer as compared with the analogous magnitude of a nonscattering layer. The inverse effect is observed for large angles Θ since the contribution of the energy scattering in the direction Θ cannot cancel the energy loss in the other directions. Therefore, the curve of the angular dependence of the spectral emissivity has a maximum in the case of a scattering layer. As the albedo ω grows, the maximum shifts towards small angles and for a certain ω the curve $\varepsilon_\nu(\Theta)$ becomes decreasing in the whole range of values of Θ ($0 \leq \Theta \leq \pi/2$) while the quantity $\varepsilon_\nu(0)$ will be less than $\varepsilon_\nu(0)$ for $\omega = 0$.

The nature of the dependence $\varepsilon_\nu(\Theta)$ for a refracting layer remains exactly as in the case n = 1, with the exception of the domain of angles $\Theta \approx \Theta_{Br}$, where an abrupt drop in the dependence $\varepsilon_\nu(\Theta)$ is observed, due to the strong influence of reflection on the boundary. However, the passage from the extremal curve to that decreasing in the whole range of angles occurs for smaller values of the optical thicknesses and albedo (Fig. 2). Results of a computation, represented in Fig. 2, permit supplementing the analysis performed in [8]. The deductions made in [8] relative to the magnitudes of the emissivities of refracting and nonrefracting media for albedo values $\omega \to 0$ are true for optically thick layers. An increase in the albedo of a refracting optically thin layer results in values of $\varepsilon_\nu(\Theta)$ being less than the $\varepsilon_\nu(\Theta)$ of a nonrefracting layer only for large angles Θ. In the domain of small values of Θ the magnitude of $\varepsilon_\nu(\Theta)$ of a layer with $n > 1$ exceeds $\varepsilon_\nu(\Theta)$ in the case n = 1, which can

TABLE 1. Magnitudes $\eta(\Theta)$, %, of an Anisotropically Scattering Layer (from left to right the values of $\tilde{\Theta}$ for the columns equal 0, 1, 2, 3, 4, 5, 6, 7)

$J(\Theta_0) = 1 + \cos\Theta_0$

n	τ	ω	$\tilde{\Theta}$							
1,0	0,5	0,25	0,48	0,47	0,45	0,44	0,43	0,40	0,35	0,02
		0,50	1,38	1,38	1,37	1,33	1,29	1,21	1,03	0,25
		0,75	3,89	3,89	3,87	3,79	3,62	3,36	2,78	1,43
		0,99	28,2	28,0	27,9	27,4	26,7	25,9	24,6	22,3
	1,0	0,25	1,09	1,09	1,09	1,05	1,01	0,89	0,75	0,06
		0,50	3,16	3,15	3,10	3,02	2,89	2,62	2,03	0,67
		0,75	8,06	8,02	7,88	7,62	7,19	6,44	5,19	3,13
		0,99	32,3	32,1	31,8	31,3	30,6	29,4	27,6	24,9
	1,5	0,25	1,64	1,64	1,62	1,58	1,49	1,32	0,97	0,15
		0,50	4,52	4,51	4,43	4,28	4,01	3,53	2,63	0,98
		0,75	10,4	10,3	10,1	9,70	9,07	7,97	6,40	4,00
		0,99	34,3	34,2	33,9	33,3	32,3	31,1	29,0	25,7
	5,0	0,25	2,98	2,95	2,85	2,68	2,40	1,96	1,31	0,26
		0,50	7,09	7,02	6,78	6,40	5,77	4,85	3,50	1,43
		0,75	13,8	13,7	13,3	12,7	11,7	10,2	8,13	5,02
		0,99	39,4	39,2	38,7	37,7	36,2	34,1	30,6	25,6
1,5	0,5	0,25	0,18	0,18	0,18	0,17	0,17	0,17	0,16	0,00
		0,50	0,59	0,59	0,59	0,59	0,59	0,59	0,58	0,00
		0,75	2,02	2,01	2,00	2,00	1,98	1,96	1,94	0,00
		0,99	23,8	23,8	23,8	23,7	23,7	23,6	23,5	0,00
	1,0	0,25	0,44	0,44	0,44	0,44	0,44	0,43	0,43	0,00
		0,50	1,41	1,41	1,39	1,39	1,38	1,37	1,35	0,00
		0,75	4,03	4,00	4,00	3,98	3,94	3,90	3,83	0,00
		0,99	30,5	30,5	30,4	30,4	30,2	30,2	30,1	0,00
	1,5	0,25	0,70	0,70	0,70	0,70	0,68	0,67	0,66	0,00
		0,50	2,06	2,06	2,05	2,04	2,01	1,99	1,95	0,00
		0,75	5,17	5,16	5,13	5,09	5,04	4,94	4,85	0,00
		0,99	35,2	35,1	35,1	35,1	34,9	34,8	34,6	0,00
	5,0	0,25	1,36	1,36	1,35	1,33	1,30	1,26	1,20	0,00
		0,50	3,30	3,29	3,26	3,22	3,15	3,06	2,97	0,00
		0,75	6,86	6,85	6,80	6,73	6,62	6,49	6,33	0,00
		0,99	49,6	49,6	49,5	49,4	49,2	49,1	48,7	0,00

$J(\Theta_0) = 1 + \cos^3\Theta_0$

n	τ	ω	$\tilde{\Theta}$							
1,0	0,5	0,25	0,30	0,30	0,30	0,30	0,30	0,30	0,28	0,02
		0,50	0,90	0,90	0,90	0,90	0,90	0,90	0,80	0,20
		0,75	2,00	2,00	2,00	2,00	2,00	2,00	2,00	0,90
		0,99	15,0	15,0	15,0	15,0	15,0	14,0	13,0	12,0
	1,0	0,25	0,70	0,70	0,70	0,70	0,70	0,60	0,60	0,02
		0,50	1,80	1,80	1,80	1,80	1,80	1,80	1,50	1,40
		0,75	4,40	4,40	4,40	4,40	4,40	4,10	3,40	1,90
		0,99	17,0	17,0	17,0	17,0	17,0	16,0	16,0	14,0
	1,5	0,25	0,90	0,90	0,90	0,90	0,90	0,70	0,70	0,10
		0,50	2,50	2,50	2,50	2,40	2,30	2,30	1,80	0,60
		0,75	5,50	5,50	5,50	5,50	5,40	5,00	4,00	2,00
		0,99	17,7	17,7	17,6	17,5	17,4	17,3	16,3	13,9
	5,0	0,25	1,60	1,60	1,60	1,60	1,50	1,30	0,90	0,66
		0,50	3,77	3,77	3,75	3,70	3,50	3,10	2,30	0,87
		0,75	7,20	7,20	7,20	7,16	6,90	6,30	5,10	2,90
		0,99	19,9	19,9	19,8	19,7	19,7	18,9	17,1	13,7
1,5	0,5	0,25	0,13	0,13	0,13	0,13	0,12	0,11	0,10	0,00
		0,50	0,46	0,46	0,44	0,43	0,41	0,38	0,34	0,00
		0,75	1,52	1,52	1,47	1,41	1,32	1,22	1,10	0,00
		0,99	13,3	13,2	13,1	12,8	12,5	12,1	11,7	0,00
	1,0	0,25	0,35	0,35	0,33	0,33	0,30	0,28	0,25	0,00
		0,50	1,08	1,08	1,03	1,00	0,94	0,84	0,80	0,00
		0,75	2,90	2,90	2,80	2,70	2,50	2,37	2,18	0,00
		0,99	16,4	16,3	16,1	15,8	15,4	15,0	14,5	0,00
	1,5	0,25	0,54	0,53	0,53	0,51	0,47	0,44	0,40	0,00
		0,50	1,55	1,55	1,50	1,44	1,36	1,25	1,14	0,00
		0,75	3,60	3,58	3,50	3,37	3,16	2,98	2,74	0,00
		0,99	18,4	18,3	18,1	17,7	17,3	16,8	16,3	0,00
	5,0	0,25	1,05	1,04	1,00	0,96	0,89	0,81	0,72	0,00
		0,50	2,40	2,39	2,33	2,22	2,07	1,90	1,72	0,00
		0,75	4,65	4,62	4,50	4,33	4,09	3,80	3,50	0,00
		0,99	24,1	24,0	23,7	23,3	22,8	22,2	21,5	0,00

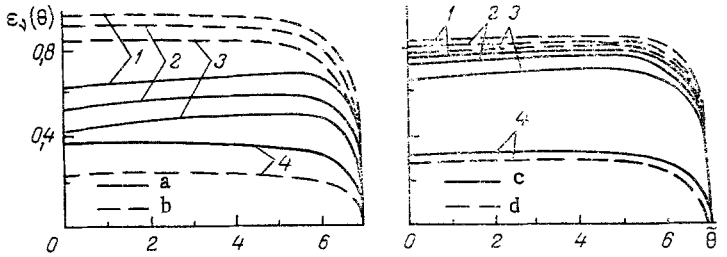

Fig. 2. Influence of the albedo (ω) and optical layer thickness (τ) on the magnitude of the spectral directional emissivity of an isotropically scattering layer with n = 1.5 (1-4 and a-d see Fig. 1).

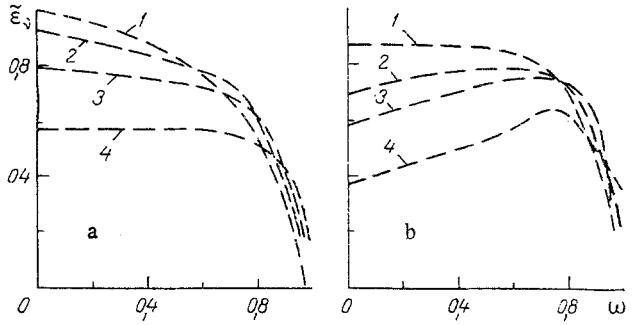

Fig. 3. Dependence of the spectral hemispherical emissivity of an isotropically scattering layer on the albedo (ω) and optical thickness (τ) of nonrefracting (a) and refracting layers with n = 1.5 (b): 1) τ = 5; 2) 1.5; 3) 1; 4) 0.5.

be explained by compensation of the internal reflection of the energy redistribution from the domain of large Θ. Further growth in the albedo is accompanied by propagation of this effect to the whole domain of angles $0 \leqq \Theta \leqq \Theta_{Br}$. In the ω range considered, the emissivity $\varepsilon_\nu(\Theta)$ of a refracting optically thick layer is greater than the $\varepsilon_\nu(\Theta)$ of a layer with n = 1 and grows as the albedo diminishes.

The influence of the optical parameters on the magnitude of the spectral hemispherical emissivity is shown in Fig. 3. The volume scattering diminishes the magnitude of the hemispherical capacity which is characteristic for all the thicknesses considered. For small albedoes ($\omega < 0.5$) higher emissivity values correspond to large optical thickness, while for $\omega > 0.5$ the dependence $\tilde{\varepsilon}_\nu(\tau)$ becomes extremal. The optical thickness at which the extremum is achieved diminishes as the albedo grows. This effect can be explained by the influence of scattering on the energy distribution in the layer [13], whereupon the greater the albedo in an optically thick medium, the greater the part of the radiation energy that remains within and does not reach the layer surface. Multiple reflection exerts influence of the hemispherical emissivity of a refractive layer in addition to the listed effects. In this case, the curve of the dependence of the change in the quantity $\tilde{\varepsilon}_\nu$ on the albedo for small optical thicknesses has an extremum that shifts toward high ω as the thickness diminishes.

Before starting to examine the results obtained for anisotropic layers, let us note that according to [14] the fraction of forward-scattered radiation can be represented in the form of the formula

$$F = (1/2)\int_0^{\pi/2} J(\Theta_0)\sin\Theta_0 d\Theta_0.$$

In the isotropic scattering case F = 0.5. For the indices (2) and (3), F will take on the values 0.75 and 0.62, respectively. This permits the assumption that the greatest contrast in the computation results for directional emissivity in an anisotropic layer from the $\varepsilon_\nu(\Theta)$ of an isotropic layer will be in the direction $\Theta \to 0$ and this difference in magnitude should appear more strongly in the case of the index (2).

Results of computations of the emissivities of anisotropic layers showed that the nature of the dependences $\varepsilon_\nu(\Theta)$ and $\tilde{\varepsilon}_\nu$ in the range of ω, τ and n under investigation did not change

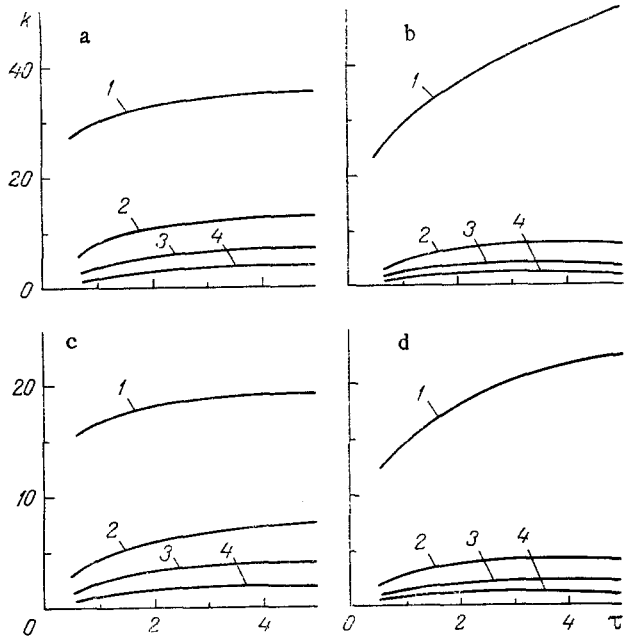

Fig. 4. Dependence of the quantity k, % on the layer optical thickness (τ) and albedo [1) ω = 0.989; 2) 0.75; 3) 0.5; 4) 0.25] in the case of anisotropic scattering with the indices $J(\Theta_0) = 1 + \cos\Theta_0$ [a) n = 1; b) 1.5] and $J(\Theta_0) = 1 + (\cos\Theta_0)^3$ [c) n = 1; d) 1.5].

as compared with an isotropically scattering layer; however, the magnitudes of $\varepsilon_\nu(\Theta)$ and ε_ν became different. As should have been expected, the curves $\eta(\Theta)$ are decreasing as the angle Θ varies between 0 and $\pi/2$ outside the dependence on ω and the kinds of index. The $\eta(\Theta)$ obtained for a nonrefracting layer with index (2) is almost twice the $\eta(\Theta)$ in magnitude in the case of scattering with the index (3). Because of multiple reflection in a layer with n = 1.5, the $\eta(\Theta)$ becomes practically linearly decreasing for both indices (see Table 1).

The change in the quantity $k(\tau)$ for different ω and n is shown in Fig. 4. For the indices (2) and (3) the dependence $k(\tau)$ is growing in the whole range of albedo and refraction coefficient variation. Moreover, the influence of ω and n on the nature of the change in the dependence $k(\tau)$ turns out to be identical for these indices. An increase in the value of the refractive coefficient results in a reduction of the difference between the hemispherical emissivities of the isotropic and anisotropic layers in the case of small ω. However, for $\omega \to 1$ a growth of $k(\tau)$ is observed for optically thick layers and a reduction of this quantity for small τ as compared with the case n = 1.

Therefore, neglecting the anisotropy of scattering can result in significant errors in calculating the emissivities of semiopaque layers. An increase in the albedo and optical thickness contributes to error growth. The influence of the refractive coefficient on the error is ambiguous and depends on the ω and τ of the layer.

NOTATION

$\varepsilon_\nu(\Theta)$, $\tilde{\varepsilon}_\nu$, spectral directional and hemispherical emissivities; d, plane layer thickness; c, velocity of electromagnetic wave propagation in a vacuum; Φ_ν^+, Φ_ν^-, spectral intensities of the thermal waves making acute angles with the internal normals to the surfaces x = 0 and x = d, respectively; α_ν, β_ν, $\varkappa_\nu$, n_ν, spectral coefficients of attenuation, volume scattering, absorption, and refraction; $J(\Theta_0)$, volume scattering index; Θ, Θ', scattering and incident ray directions; Θ_0, angle between the incident and scattered rays; Θ_{Br}, Brewster angle; R_1, R_2, reflection coefficients from the external and internal surfaces of the layer; ν, emission frequency; $B_\nu(T)$, Planck function; $\Phi_{\nu,e}^+$, $\Phi_{\nu,e}^-$, intensities of the external emission incident on the left and right surfaces of the layer; $\varepsilon_\nu{}^1(\theta)$, $\tilde{\varepsilon}_\nu{}^1$, spectral directional and hemispherical emissivities of an anisotropically scattering layer; $\varepsilon_\nu{}^0(\theta)$, $\tilde{\varepsilon}_\nu{}^0$, spectral directional and hemispherical emissivities of an isotropically scattering layer; $\omega = (\varkappa_\nu + \beta_\nu)/\beta_\nu$, albedo of volume scattering; $\tilde{\Theta} = \Theta/\Delta\Theta$, $\Delta\Theta = (\arcsin(1/n))/7$; $\tau = d\varkappa_\nu$, optical thickness of the layer.

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NONEQUILIBRIUM COUNTERFLOW CAPILLARY IMPREGNATION

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A model of capillary impregnation, taking account of disequilibrium of the phase permeabilities, is constructed on the basis of a general scheme of nonequilibrium two-phase filtration proposed in [3]; see also [2].

The theory of counterflow capillary impregnation of a porous medium is constantly under examination by researchers, in particular, in connection with the role played by this process in the displacement of petroleum by water in microinhomogeneous hydrophilic beds. The existing model of capillary impregnation is based on the self-similar solution of [1] (see also [2]), using the classical Muskat–Leverette scheme of the filtration of inhomogeneous liquids. According to this scheme, the relative phase permeabilities of water and petroleum and also the reduced capillary pressure (the Leverette function) are regarded as universal functions of instantaneous saturation σ, which may be determined from data on the steady flow of a mixture of the given composition. However, in reality, the characteristic impregnation time in low-permeability microinhomogeneous blocks may be comparable with the time to establish phase permeabilities and capillary pressure, i.e., the time for regrouping of the liquids along channels of the appropriate dimensions. For this reason, the model of counterflow capillary impregnation must take account of disequilibrium effects.

1. Basic Equations of Model

For the combined filtration of water and petroleum, under broad assumptions, Darcy's law is valid

$$\mathbf{u}_1 = -(k/\mu_1) f_1 \nabla p_1, \quad \mathbf{u}_2 = -(k/\mu_2) f_2 \nabla p_2,$$
$$p_2 - p_1 = T \cos\Theta (m/k)^{1/2} J. \tag{1}$$

The simplest formulation of the scheme for taking account of disequilibrium [3, 2] rests on the basis that the functions f_1, f_2, J determined from the data on steady flow of the mixture are monotonic functions of the true water saturation σ. The functions f_1, f_2 vary here

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